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THE USE OF THE RADICAL SYMBOL.

By G. A. Miller.

Probably the most significant recent step towards the uniformisation of the notation of elementary and secondary mathematics is represented by the list of symbols, numbered from 1 to 139, proposed in Heft 17 of the *Schriften des Deutschen Ausschusses für den mathematischen und naturwissenschaftlichen Unterricht,* 1913. In many cases remarks and explanations are added to the proposed symbols. Number 20 of these symbols relates to an *n*th root, and, as this may be of special interest, we shall reproduce it here, translated into English.

20 $\sqrt[n]{a}$ *n*th root of *a*.

Remark 1.—The use of the parenthesis in place of the stroke is to be avoided when possible.

Remark 2.—The notation $\sqrt{}a$ without the stroke is allowed when the root symbol is followed by a number represented by Hindu-Arabic numerals or by a single letter.

Remark 3.—The root index should be written within the root symbol. Thus $\sqrt[n]{a}$, not $^n\sqrt{a}$.

Remark 4.—In case of the square root the index is commonly omitted.

Remark 5.—The $\sqrt[n]{a}$ represents the positive number whose *n*th power is *a* when *a* is positive. When *a* is negative and *n* is odd then $\sqrt[n]{a}$ represents the real number whose *n*th power is *a*.

Explanation.—The single-valuedness of the root symbol noted in Remark 5 seems to offer the only possibility of avoiding inexactness in teaching students in the schools. It is worth noting that the equations

$$a^3 = 1 \text{ and } a = \sqrt[3]{1}$$

as well as the equations

$$a^2 = 1 \text{ and } a = \sqrt{1}$$

are not regarded as identical according to the said remark. The former equation in each of these two sets of equations is for the time being to be regarded as multiply-valued, while the latter is single-valued. The confusion is especially serious in the introduction of logarithms, if we let

$$\sqrt[n]{a} = a^{1/n}$$

and then transform

$$a^{1/n} = b \text{ into } \frac{1}{n} = \log_a b$$

without having first stated the single-valuedness of $\sqrt[n]{a}$.

While the said restrictions on the meaning of the radical sign tend to definiteness they evidently do not touch some of the most serious difficulties. Even in our elementary algebra courses we are concerned with square root of negative numbers. For instance, the roots of the quadratic equation

$$ax^2 + bx + c = 0$$

are commonly represented in the form

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The question arises what does

$$\sqrt{b^2 - 4ac}$$

mean when $b^2 - 4ac$ is negative or complex. We should clearly not speak of a positive and a negative square root in this case since the terms positive and negative relate only to real numbers, being short expressions for argument of 0° and 180° respectively. On the other hand, it is customary to consider $\sqrt{-1} = i$ as situated on the positive part of the y-axis, and it would appear desirable to add to Remark 5, noted above, that $\sqrt[n]{a}$, when a is negative and n is even, represents the number whose argument is π/n and whose nth power is a.

If this were done the symbol $\sqrt[n]{a}$ would have a unique meaning whenever a is real, and n is a positive integer, and the students of elementary algebra could be taught a single meaning of this symbol which would suffice for the solution of the quadratic equation with real coefficients. It is true that this definition of the symbol $\sqrt[n]{a}$ has the disadvantage of lacking generality, since when a is negative we define it in two different ways as n is odd or even, but if the difference of these definitions were properly emphasized it would tend to clearness in the use of the radical sign in our elementary work.

The question whether it would be desirable to define the symbol $\sqrt[n]{a}$ uniquely also when a is any complex number is not pressing so much for an answer. In fact, desirable definiteness would be secured by stating that this symbol has a unique meaning only when n is a positive integer and a is real. When a is complex it would clearly be possible to define $\sqrt[n]{a}$ uniquely by saying that it represents the number whose argument is $1/n$th the argument of a and whose nth power is a, and thus secure uniformity with the case when a is negative and n even. With such restrictions on the radical sign it would have distinct advantages over the corresponding fractional exponent notation and there would appear substantial reasons for continuing the use of the radical sign in elementary algebra.

To avoid the possibility of an incorrect inference it is desirable to add that in the "explanation" quoted above we represented log b to the base a in the usual form $\log_a b$. In said list this is written in the form

$${}^{a}\log b$$

in accord with a notation suggested by A. L. Crelle in his "Sammlung mathematische Aufsätze," Vol. 1, 1821, page 207, and it is suggested that the base should always be noted unless it is 10 or e. In many German publications the base is written above the abbreviation of the word logarithms, thus

$$\overset{a}{\log}\, b.$$

The form $\log_a b$ is commonly used not only in English, French, and Italian, but also to some extent in German, and hence it would appear that its use should become universal. It seems unfortunate that this form does not appear in said list of suggested symbols.

It should perhaps also be explicitly noted that the logarithm of every number has really an infinite number of values as was proved already by L. Euler. In using logarithms of positive numbers it is customary in elementary mathematics to confine our attention to the single real value. In this way $\log a$ becomes a single-valued function of a whenever a is a positive number. The given suggestions as regards the single value of the radical are therefore in accord with other restrictions in our early mathematical work, and such suggestions have been advocated many times. In fact, in his noted "Analyse algébrique," 1821, A. L. Cauchy distinguished for the first time the principal values of elementary functions by a convenient notation, using the symbols

$$l(z), \quad a^z, \quad \sqrt[n]{\ } \qquad \sin z$$

for the principal values, and the symbols

$$l((z)), \quad ((a))^z, \quad \sqrt[n]{\!\sqrt{a}}, \quad \text{arc sin}\ ((z))$$

for the corresponding general values of these expressions.* This notation has, however, not been widely adopted and we still find too frequently the same notation used both for the principal value and for the general values.

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* "Encyclopédie des Sciences Mathématiques," tome II, Vol. 2, p. 50.